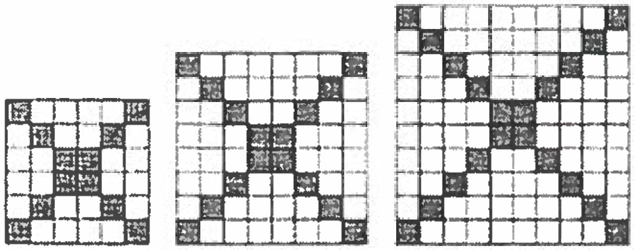
KEY

The diagrams below represent the first three terms of a sequence.



Term 1 Term 2 Term 3

12 16 20

$a_1 = 12$

$a_2 = 16$

$a_3 = 20$

Assuming the pattern continues, which formula determines a_n, the number of shaded squares in the nth term?

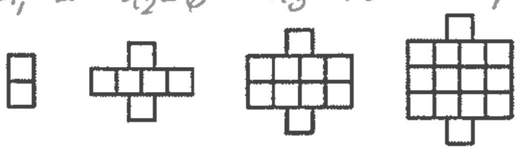
1) $a_n = 4n + 12$ ×
2) $a_n = 4n + 8$ ✓
3) $a_n = 4n + 4$
4) $a_n = 4n + 2$

$a_1 = 4(1) + 12$
× $a_1 = 16$ ×

$a_1 = 4(1)+12$	$a_2 = 4(2)+8$	$a_3 = 4(3)+8$
$a_1 = 16$ ✓	$a_2 = 16$ ✓	$a_3 = 20$ ✓

A pattern of blocks is shown below.

$a_1 = 2$ $a_2 = 6$ $a_3 = 10$ $a_4 = 14$



Term 1 Term 2 Term 3 Term 4

If the pattern of blocks continues, which formula(s) could be used to determine the number of blocks in the nth term?

I ×	II ✓	III ✓
$a_n = n + 4$	$a_1 = 2$ $a_n = a_{n-1} + 4$	$a_n = 4n - 2$

1) I and II
2) I and III
3) II and III
4) III, only

I	II		III
$a_1 = 1 + 4$ × $a_1 = 5$ ×	$a_1 = 2$ ✓ $a_2 = a_{2-1} + 4$ $= a_1 + 4$ $= 2 + 4$ $a_2 = 6$ ✓	$a_3 = a_{3-1} + 4$ $= a_2 + 4$ $= 6 + 4$ $a_3 = 10$ ✓	$a_1 = 4(1) - 2$ $a_1 = 2$ ✓ $a_2 = 4(2) - 2$ $a_2 = 6$ ✓ $a_3 = 4(3) - 2$ $a_3 = 10$ ✓

What is the common difference of the arithmetic sequence 5,8,11,14?

$+3 \; +3 \; +3$

1) $\frac{8}{5}$
2) -3
3) 3
4) 9

Is the sequence arithmetic? If so, find the common difference.

$-2.4, 6.4, 15.2, 24$

$+8.8 \; +8.8 \; +8.8$

[A] no [B] yes, 8.8

[C] yes, 8.9 [D] yes, 8.6

Which arithmetic sequence has a common difference of 4?

1) ~~$\{0,4n,8n,12n,\ldots\}$~~ → 4n = common diff
2) ~~$\{n,4n,16n,64n,\ldots\}$~~ → common ratio = 4
3) $\{n+1,n+5,n+9,n+13,\ldots\}$
4) ~~$\{n+4,n+16,n+64,n+256,\ldots\}$~~

Is the sequence arithmetic? If so, find the common difference.

$-6.9, -10, -13.1, -16.2$

$-3.1 \; -3.1 \; -3.1$

[A] yes, −3.3 [B] yes, −3

[C] no [D] yes, −3.1

The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_1, which is an equation for the nth term of this sequence?

1) $a_n = 8n + 10$
2) $a_n = 8n - 14$
3) $a_n = 16n + 10$
4) $a_n = 16n - 38$

$a_1 = -6$ +8
$a_2 = 2$ +8
$a_3 = 10$
$a_4 = 18$ +8
$a_5 = 26$ +8
+16

Is the sequence arithmetic? If so, find the common difference.

11.1, 21.6, 32.2, 42.6

+10.5 +10.6 +10.4

[A] yes, 10.4 [B] no

[C] yes, 10.7 [D] yes, 10.5

In an arithmetic sequence, $a_4 = 19$ and $a_7 = 31$. Determine a formula for a_n, the n^{th} term of this sequence.

$\boxed{a_n = 4n+3}$

$a_4 = 19$
$a_5 = 23$ +4
$a_6 = 27$ +4
$a_7 = 31$ +4
+12

Which recursively defined function has a first term equal to 10 and a common difference of 4?

+4

1) $f(1) = 10$
   $f(x) = f(x-1) + 4$
2) ~~$f(1) = 4$~~
   $f(x) = f(x-1) + 10$
3) $f(1) = 10$
   $f(x) = 4f(x-1)$
4) ~~$f(1) = 4$~~
   $f(x) = 10f(x-1)$

What is a formula for the nth term of sequence B shown below?

$$B = 10, 12, 14, 16, \ldots$$

+2 +2 +2

1) $b_n = 8 + 2n$
2) $b_n = 10 + 2n$
3) $b_n = 10(2)^n$
4) $b_n = 10(2)^{n-1}$

$b_n = 10 + (n-1)2$
$b_n = 10 + 2n - 2$
$b_n = 8 + 2n$

A theater has 35 seats in the first row. Each row has four more seats than the row before it. Which expression represents the number of seats in the nth row?

1) $35 + (n + 4)$
2) $35 + (4n)$
3) $35 + (n + 1)(4)$
4) $35 + (n - 1)(4)$

$a_1 = 7$
$a_2 = 11$
$a_3 = 15$
$a_4 = 19$
$a_5 = 23$ +4
$a_6 = 27$ +4
$a_7 = 31$ +4
+12

$a_n = a_1 + (n-1)d$
$a_n = 7 + (n-1)4$
$a_n = 7 + 4n - 4$
$\boxed{a_n = 4n + 3}$

common diff = 8
$a_n = a_1 + (n-1)d$
$a_n = -6 + (n-1)8$
$a_n = -6 + 8n - 8$
$a_n = 8n - 14$

KEY

SEQUENCES

A rose is 4 inches tall at week 0 and grows 3 inches each week. Which function(s) shown below can be used to determine the height, $f(n)$, of the rose in n weeks?

$a_0 = 4$
$a_1 = 7$) +3
$a_2 = 10$) +3
$a_3 = 13$

~~I.~~ $f(n) = 4n + 3(n - 1)$
✓II. $f(n) = 3n + 4$
✓III. $f(n) = f(n - 1) + 3$ where $f(0) = 4$

I
$f(2)=4(2)+3(2-1)$
$f(2)=8+3(1)$
$f(2)=11$

1) II only

3) I and II only

2) I and III only

4) II and III only

II
$f(2)=3(2)+4$
$f(2)=10$ ✓
$f(3)=3(3)+4$
$f(3)=13$ ✓

III
$f(2)=f(2-1)+3$
$f(2)=f(1)+3$
$f(2)=7+3$
$f(2)=10$ ✓

$f(3)=f(3-1)+3$
$f(3)=f(2)+3$
$f(3)=10+3$
$f(3)=13$ ✓

2. The fourth term in an arithmetic sequence is 16 and the sixth term is 40. If the first term is a_1, which is an equation for the n^{th} term of this sequence?

$a_1 = -20$
$a_2 = -8$
$a_3 = 4$
$a_4 = 16$) +12
$a_5 = 28$) +12 } +24
$a_6 = 40$

1) $a_n = 12n + 32$

3) $a_n = 32n - 12$

2) $a_n = 12n - 32$

4) $a_n = 32n + 12$

$a_n = a_1 + (n-1)d$
$a_n = -20 + (n-1)12$
$a_n = -20 + 12n - 12$
$a_n = -32 + 12n$

3. The third term in an arithmetic sequence is 15 and the sixth term is 27. If the first term is a_1, which is an equation for the n^{th} term of this sequence?

1) $a_n = 4n + 3$

3) $a_n = 3n + 4$

2) $a_n = 4n - 3$

4) $a_n = 3n - 4$

$a_1 = 7$
$a_2 = 11$
$a_3 = 15$
$a_4 = 19$) +4
$a_5 = 23$) +4
$a_6 = 27$) +4
12

$a_n = a_1 + (n-1)d$
$a_n = 7 + (n-1)4$
$a_n = 7 + 4n - 4$
$a_n = 3 + 4n$

KEY

Name____________________________ Date ________________

Recursive Sequences Algebra I

Recursion is the process of choosing a starting term and repeatedly applying the same process to each term to arrive at the following term.

A **recursive formula** always has two parts:

1. the starting value for a_1
2. the recursion equation for a_n as a function of a_{n-1}

Regents Questions:

Which recursively defined function has a first term equal to 10 and a common difference of 4?

$a_1 = 10$

(1) $f(1) = 10$
$f(x) = f(x-1) + 4$

(2) $f(1) = 4$
$f(x) = f(x-1) + 10$

(3) $f(1) = 10$
$f(x) = 4f(x-1)$

(4) $f(1) = 4$
$f(x) = 10f(x-1)$

Which recursively defined function represents the sequence 3, 7, 15, 31, ...?

f(1) f(2) f(3)

(1) $f(1) = 3,\ f(n+1) = 2^{f(n)} + 3$

$f(1+1) = 2^{f(1)} + 3$
$f(2) = 2^3 + 3$
$\times$ $f(2) = 11$

(2) $f(1) = 3,\ f(n+1) = 2^{f(n)} - 1$

$f(1+1) = 2^{f(1)} - 1$ $f(2+1) = 2^{f(2)} - 1$
$f(2) = 2^3 - 1$ $f(3) = 2^7 - 1$
$f(2) = 7$ ✓ $f(3) = 127$ $\times$

(3) $f(1) = 3,\ f(n+1) = 2f(n) + 1$

$f(1+1) = 2f(1) + 1$ $f(2+1) = 2f(2) +$
$f(2) = 2(3) + 1$ $f(3) = 2(7) + 1$
$f(2) = 7$ ✓ $f(3) = 15$ ✓

$f(3+1) = 2f(3) + 1$
$f(4) = 2(15) + 1$
$f(4) = 31$ ✓

(4) $f(1) = 3,\ f(n+1) = 3f(n) - 2$

If a sequence is defined recursively by $f(0) = 2$ and $f(n+1) = -2f(n) + 3$ for $n \geq 0$, then $f(2)$ is equal to

(1) 1
(2) -11
(3) 5
(4) 17

$f(0+1) = -2f(0) + 3$
$f(1) = -2(2) + 3$
$f(1) = -1$

$f(1+1) = -2f(1) + 3$
$f(2) = -2(-1) + 3$
$f(2) = 5$

KEY

Name________________________________ Date ________________

Arithmetic Sequences Algebra I

- A **sequence** is an ordered list of numbers.
- While some sequences are simply random values, other sequences have a **definite pattern**. One such sequence is an **arithmetic sequence**.
- If the sequence of values follows a pattern of **adding a fixed amount** from one term to the next, it is referred to as an arithmetic sequence.
- The fixed amount is called the **common difference, *d***.

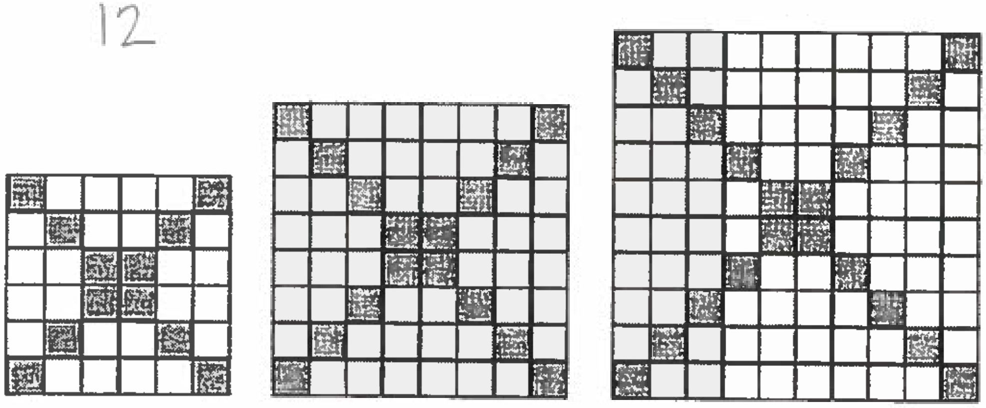
To find any term of an arithmetic sequence:

$$a_n = a_1 + (n - 1)d$$

Regents Questions:

The diagrams below represent the first three terms of a sequence.

12



Term 1 Term 2 Term 3

$a_1 = 12$ $a_2 = 16$ $a_3 = 20$

common diff = 4

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(4) $a_n = 4n + 2$

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$a_n = 12 + 4n - 4$
$a_n = 8 + 4n$

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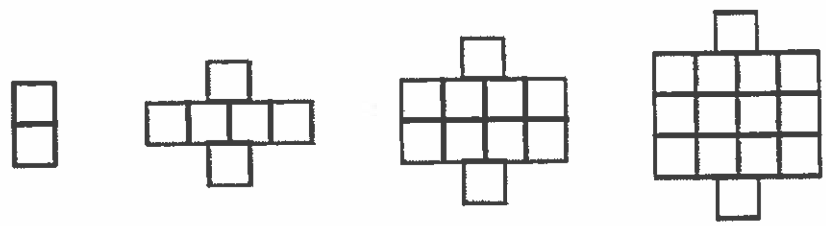
$a_1 = -6$ $a_2 = 2$
$a_3 = 10$ $)+8$
$a_4 = 18$ $)+8$ $+16$
$a_5 = 26$

(1) $a_n = 8n + 10$
(2) $a_n = 8n - 14$
(3) $a_n = 16n + 10$
(4) $a_n = 16n - 38$

$a_n = -6 + (n-1)8$
$a_n = -6 + 8n - 8$
$a_n = -14 + 8n$

A pattern of blocks is shown below.



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If the pattern of blocks continues, which formula(s) could be used to determine the number of blocks in the nth term?

I	II	III
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(1) I and II
(2) I and III
(3) II and III
(4) III, only

$a_n = a_1 + (n-1)d$
$a_n = 2 + (n-1)4$
$a_n = 2 + 4n - 4$
$a_n = 4n - 2$