Name Answer Key

REVIEW – Functions

Date

Algebra CC

1) After a car is purchased, it depreciates (loses) in value. If a pickup truck costs $45,500 and it depreciates 4% per year, how much is the car worth after 3 and ½ years?

$y = a(1-r)^t$

$y = 45{,}500(1-0.04)^{3.5}$

$y = 45{,}500(0.96)^{3.5}$

$y = 39442.16198$

$y \approx \$39442.16$

2) A mouse population starts with 2,000 mice and grows at a rate of 5% per year. To the nearest whole number, find the population of the mice after 5 years.

$y = a(1+r)^t$

$y = 2000(1+0.05)^5$

$y = 2000(1.05)^5$

$y = 2552.563125$

$y \approx 2{,}553$ mice

3) Labor at the car repair shop can be represented by the function:

$$\text{Total charge for repairs} = \begin{cases} 150, & 0 < h \le 1 \\ 150 + 80(h-1), & h > 1 \end{cases}$$

If $h$ represents the number of hours worked, what is the charge for a 3 hour repair?

$150 + 80(h-1)$

$150 + 80(3-1)$

$150 + 80(2)$

$150 + 160$

$\$310$

a 4) Joseph's tax charges $10.00 for the initial service of any drive. Then, the fee for each mile $0.75. Which type of function is represented by this situation?

a) Linear b) Exponential c) quadratic d) absolute value

5) Calculate $f(-4)$ for the function: $f(x) = 3x^2 + 4x - 2$

$f(-4) = 3(-4)^2 + 4(-4) - 2$

$f(-4) = 3(16) - 16 - 2$

$f(-4) = 48 - 16 - 2$

$f(-4) = 30$

6) Jeff bought a new car for $10,450. He knows this car's value will decrease by 20% each year.

a) Write a function to model the cost of his car after $n$ years.

b) If Jeff plans to sell the car after five years, what will be the value of the car at that time, to the *nearest dollar*?

a) $y = a(1-r)^t$

$y = 10{,}450(1-0.20)^n$

$y = 10{,}450(0.80)^n$

b) $y = 10{,}450(0.80)^n$

$y = 10{,}450(0.80)^5$

$y = 3424.256$

$y \approx \$3424.$

d 7) What is the **range** for an absolute value function?

a) $y \ge 0$ b) $y \le 0$ c) $x < 0$ d) all real #s

$y = |x|$ $y = -|x|$

d 8) A small country in Europe has been experiencing population growth that can be modeled by the equation $y = 120{,}000(1.042)^x$, where $y$ is the population of the country and $x$ is the number of years since 2010. Based on the equation, what is the percent change in the population of the country each year?

a) 1.042% b) 104.2% c) 0.042% d) 4.2%

$1 + r = 1.042$

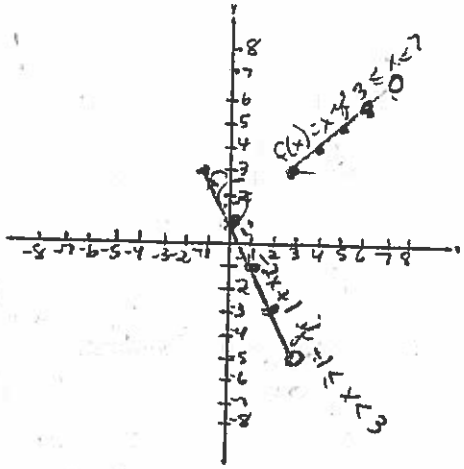
$-1 \quad -1$

$r = 0.042$

$4.2\%$

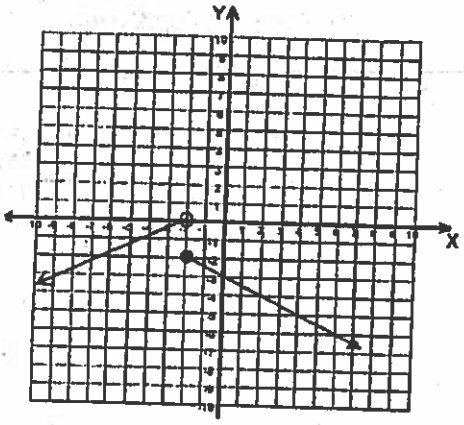
9) Graph the following piecewise function:

$$f(x) = \begin{cases} -2x + 1, if -1 \leq x < 3 \\ x, \quad if\ 3 \leq x < 7 \end{cases}$$



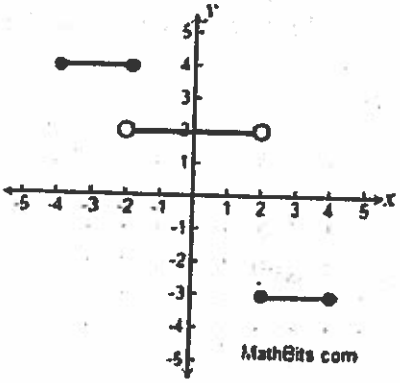
10) Write the piecewise function for the graph:

$$f(x) = y = \begin{cases} \frac{1}{2}x + 1, & \text{if } x < -2 \\ -\frac{1}{2}x - 3, & \text{if } x \geq -2 \end{cases}$$



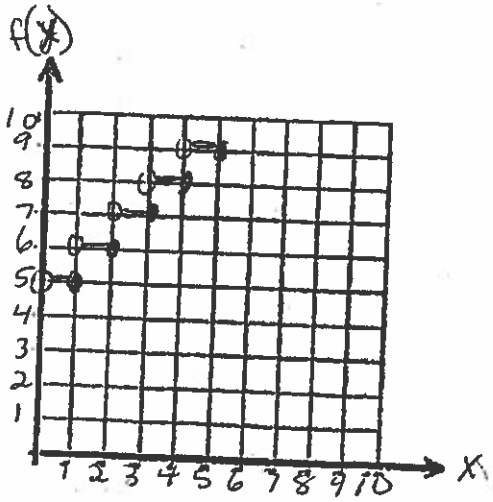
11) Write the piecewise function for the graph:

$$f(x) = y = \begin{cases} 4 & \text{if } -4 \leq x \leq -2 \\ 2 & \text{if } -2 < x < 2 \\ -3 & \text{if } 2 \leq x \leq 4 \end{cases}$$




12) Graph the following step function:

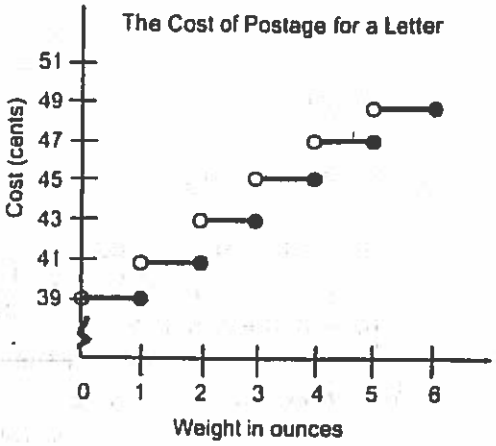
$$f(x) = \begin{cases} 5, if\ 0 < x \leq 1 \\ 6, if\ 1 < x \leq 2 \\ 7, if\ 2 < x \leq 3 \\ 8, if\ 3 < x \leq 4 \\ 9, if\ 4 < x \leq 5 \end{cases}$$



13) The cost of postage, $C(x)$, for a letter is dependent upon the weight in ounces, $x$, of the letter.

a) Complete the table below:

| Weight (oz) | 1 | 2.3 | 4.9 | 5 |
|---|---|---|---|---|
| Cost (¢) | 39¢ | 43¢ | 47¢ | 47¢ |




b) Write the function using:

1) Piecewise Notation

$$C(x) = \begin{cases} 39 \text{ if } 0 < x \leq 1 \\ 41 \text{ if } 1 < x \leq 2 \\ 43 \text{ if } 2 < x \leq 3 \\ 45 \text{ if } 3 < x \leq 4 \\ 47 \text{ if } 4 < x \leq 5 \\ 49 \text{ if } 5 < x \leq 6 \end{cases}$$

2) Step Notation

14) A cell phone plan charges by the number of minutes of usage, as shown in the chart.
- $10 for the first 30 minutes.
- increases of $2 for each 10 minute interval thereafter.

a) What is the missing value of the function?

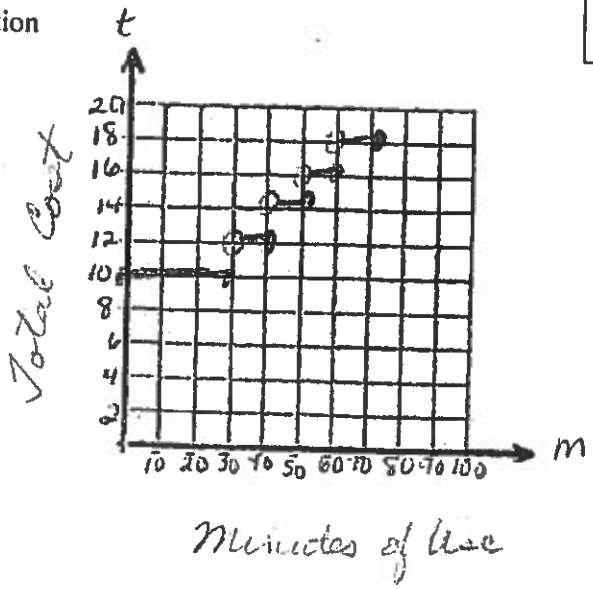
$14

| Minutes of Use, $m$ | Total Cost |
|---|---|
| $0 \leq m \leq 30$ | $10 |
| $30 < m \leq 40$ | $12 |
| $40 < m \leq 50$ | $14 |
| $50 < m \leq 60$ | $16 |
| $60 < m \leq 70$ | $18 |

2 b) What type of function should be used to depict this plan, based on minutes of use?

(1) Absolute value Function
(2) Step Function
(3) Exponential Function
(4) Linear Function

c) Graph this function



**8 Algebra CC**

**Review (Unit 12 – Exponential Functions)**

**Important Terminology**

| | | |
|---|---|---|
| exponential function | initial value | common ratio (growth factor) |
| interval | exponential growth | exponential decay |
| rate of increase/decrease | parent function | transformation |

**Exponential Function:** $y = ab^x$ **Growth Model:** $y = a(1 + r)^t$ **Decay Model:** $y = a(1 - r)^t$

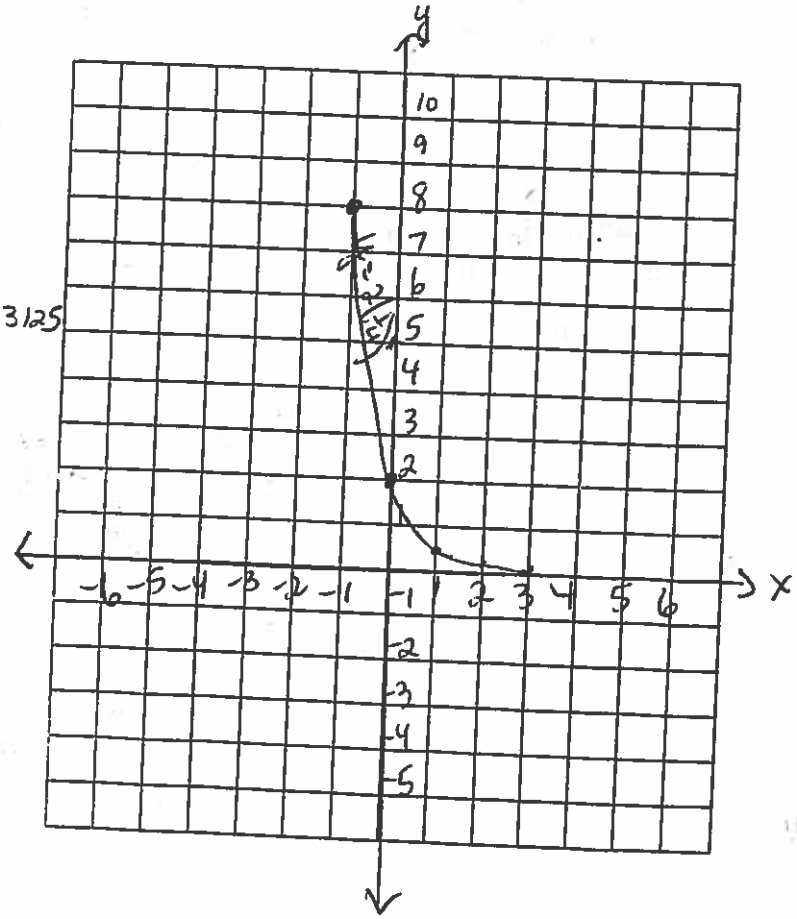
**What should I be able to do?**

1. Graph exponential functions over an interval.
2. Sketch exponential functions using a specified window range.
3. Determine if exponential functions are increasing or decreasing.
4. Identify the y-intercept of an exponential function from an equation.
5. Determine the average rate of change of an exponential function over an interval.
6. Model situations/relationships with exponential functions.
7. Solve word problems using an exponential growth or decay model.
8. Based on a parent function, identify the change that occurs when the parent function undergoes a transformation.

**Practice Problems**

1. Graph the exponential function $y = 2(\frac{1}{4})^x$ over the interval $-1 \leq x \leq 3$.

| x | $y = 2(\frac{1}{4})^x$ | y |
|---|---|---|
| −1 | $2(\frac{1}{4})^{-1}$ | 8 |
| 0 | $2(\frac{1}{4})^0$ | 2 |
| 1 | $2(\frac{1}{4})^1$ | $\frac{1}{2}$ |
| 2 | $2(\frac{1}{4})^2$ | $\frac{1}{8}$ |
| 3 | $2(\frac{1}{4})^3$ | $\frac{1}{32}$ 0.03125 |




*Remember to label everything!*

2. Identify the y-intercept of each function below and state whether the function will increase or decrease when graphed.

a) $f(x) = 34(2.75)^x$

y intercept = 34

increase

b) $f(x) = (0.25)^x$

y intercept = 1

decrease

c) $f(x) = \frac{2}{3}\left(\frac{3}{2}\right)^x$ (1.5)

y intercept = $\frac{2}{3}$

increase

3. a) Consider the exponential function $f(x) = \frac{1}{2}(4)^x$.

b) What is the average rate of change of the function over the interval $1 \le x \le 3$?

$f(x) = \frac{1}{2}(4)^x$ $\quad$ $f(x) = \frac{1}{2}(4)^x$

$f(1) = \frac{1}{2}(4)^1$ $\quad$ $f(3) = \frac{1}{2}(4)^3$

$f(1) = 2$ $\quad$ $f(3) = \frac{1}{2}(64)$

$(1, 2)$ $\quad$ $f(3) = 32$

$(3, 32)$

$slope = \frac{y_2 - y_1}{x_2 - x_1}$

$slope = \frac{32 - 2}{3 - 1}$

$slope = \frac{30}{2}$

$\boxed{slope = 15}$

c) Is the average rate of change found in part b greater than or less than the average rate of change of the function $g(x) = \frac{1}{4}(2)^x$ over the interval $4 \le x \le 8$? Justify your response.

$g(x) = \frac{1}{4}(2)^x$ $\quad$ $g(x) = \frac{1}{4}(2)^x$

$g(4) = \frac{1}{4}(2)^4$ $\quad$ $g(8) = \frac{1}{4}(2)^8$

$g(4) = \frac{1}{4}(16)$ $\quad$ $g(8) = \frac{1}{4}(256)$

$g(4) = 4$ $\quad$ $g(8) = 64$

$(4, 4)$ $\quad$ $(8, 64)$

$slope = \frac{y_2 - y_1}{x_2 - x_1}$

$slope = \frac{64 - 4}{8 - 4}$

$slope = \frac{60}{4}$

$\boxed{slope = 15}$

the same

4. A pharmaceutical company has tested a new time-release cold pill. It finds that the amount of milligrams, ***f(n)***, of the active ingredients of the pill left in the bloodstream ***n*** hours after it is taken can be estimated using the function $f(n) = 35(0.87)^n$.

a) How many milligrams of cold medicine are in the pill?

35 milligrams

b) What percent of the drug leaves the body each hour?

13%

$1 - r = 0.87$

$-1 \quad -1$

$-r = -0.13$

$r = 0.13$

c) How many milligrams, *to the nearest thousandth*, of the cold medicine remain in the body after 5 hours have passed?

$f(n) = 35(0.87)^n$

$f(5) = 35(0.87)^5$

$f(5) = 17.444\,73222$

$\boxed{f(5) \approx 17.445 \text{ mg}}$

5. Find the balance after 5 years of an account that pays 5.2% interest compounded yearly with an initial investment of $1250.

$y = a(1 + r)^t$

$y = 1250(1 + 0.052)^5$

$y = 1250(1.052)^5$

$y = 1610.603773$

$\boxed{y \approx \$1610.60}$

6. Between 1990 and 2000, the profits of a business decreased approximately 0.7% each year. In 1990, the business's profit was $1.4 million. What was the profit in 1996?

$y = a(1-r)^t$
$y = 1,400,000(1-0.007)^6$
$y = 1,400,000(0.993)^6$
$y = 1342219.446$
$y \approx 1342219.45$

7. A construction company purchased some equipment costing $300,000. The value of the equipment depreciates at a rate of 14% per year.

a) Write a formula that models the value of the equipment each year.

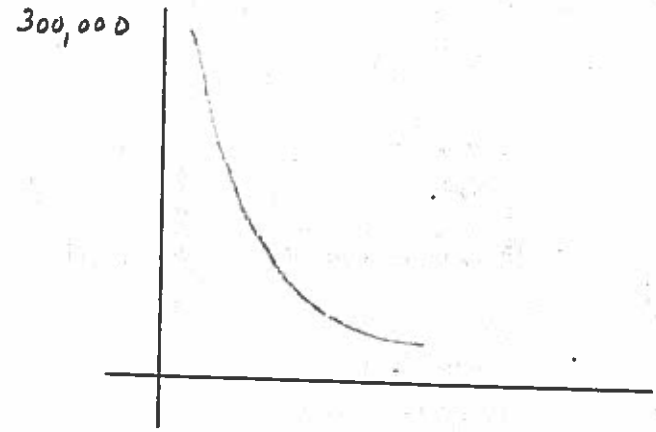
$y = a(1-r)^t$
$y = 300,000(1-0.14)^t$
$y = 300,000(0.86)^t$

b) What is the value of the equipment after 9 years?

$y = 300,000(0.86)^t$
$y = 300,000(0.86)^9$
$y = 77198.22519$
$y \approx 77198.23$

c) Sketch this function using the indicated window.
Label axes and indicate x and y maximum and minimums.

x-min: 0
x-max: 10
xscl: 1
y-min: 70,000
y-max: 300,000
yscl: 25,000




d) Estimate when the equipment will have a value of $50,000.
*Hint: access the table of values on your calculator*

a little more than 12 years

8. According to the international Basketball Association (FIBA), a basketball must be inflated to a pressure such that, when it is dropped from a height of 1,600 mm, it will rebound to a height of 1,200 mm. Maddie decides to test the rebound ability of her new basketball. She assumes that the ratio of each rebound height to the previous rebound height remains the same at $\frac{1200}{1600}$. Let ***f(n)*** be the height of the basketball after ***n*** bounces.

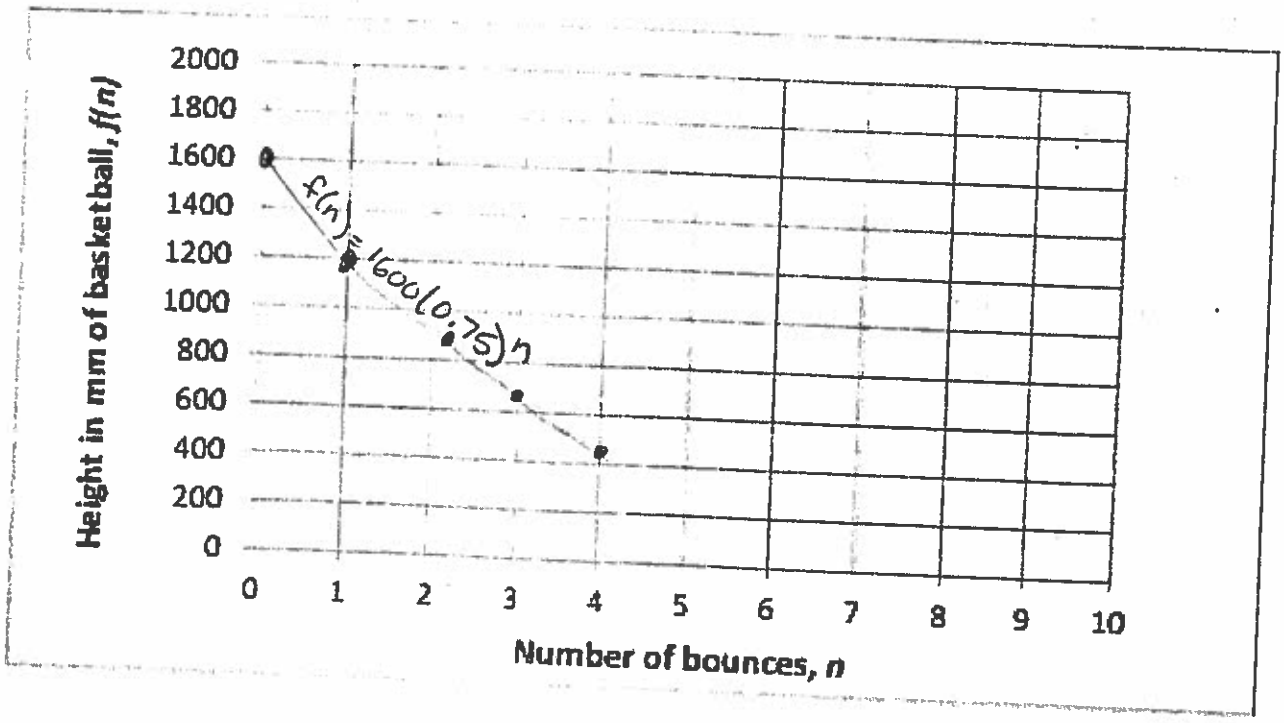
a) Complete the chart below to reflect the heights Maddie expects to measure.

| *n* | *f(n)* |
|---|---|
| 0 | 1600 |
| 1 | 1200 |
| 2 | 900 |
| 3 | 675 |
| 4 | 506.25 |

b) Write an exponential function that models this situation.

$$f(n) = 1600(0.75)^n$$

c) Graph the function on the grid below from 0 to 10 bounces. Using the curve created, estimate the bounce number at which the rebound height will drop below 200 mm.



9. Gary says that the linear function **f(x) = 2x + 4** is the same as the exponential function **g(x) = 4(2)^x**. Hel Gary understand the difference between linear functions and exponential functions by comparing and contrasting f(x) and g(x). Support your response with mathematical evidence (tables, graphs, rates of change, etc...).

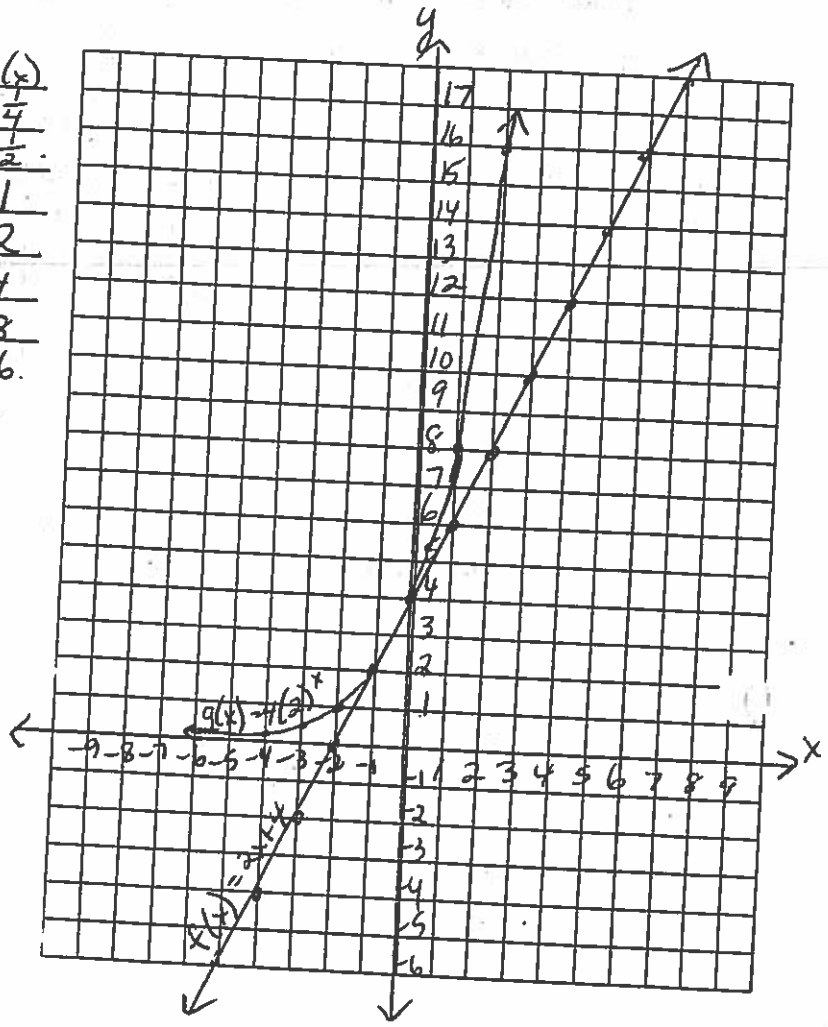
$f(x) = 2x+4$

slope $= \frac{2}{1}$

y intercept $= 4$

$g(x) = 4(2)^x$

| x | $g(x) = 4(2)^x$ | g(x) |
|---|---|---|
| -4 | $4(2)^{-4}$ | $\frac{1}{4}$ |
| -3 | $4(2)^{-3}$ | $\frac{1}{2}$ |
| -2 | $4(2)^{-2}$ | 1 |
| -1 | $4(2)^{-1}$ | 2 |
| 0 | $4(2)^{0}$ | 4 |
| 1 | $4(2)^{1}$ | 8 |
| 2 | $4(2)^{2}$ | 16 |




4 10. Which statement is ***not*** true? Use your calculator and graph

1) The graph of $y = 3^x$ and $y = -(3)^x$ have opposite y-intercepts.

2) The graph of $y = 3^x$ is a reflection of $y = -(3)^x$ in the x-axis.

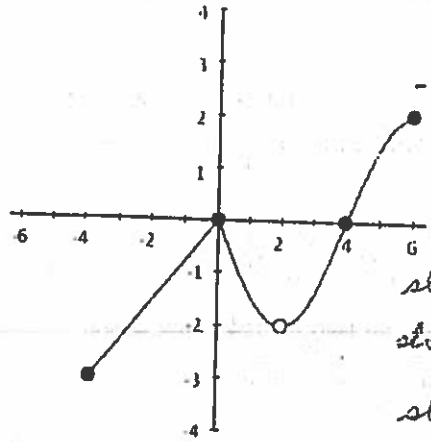
3) The graph of $y = 3^x$ is an increasing graph and $y = -(3)^x$ is a decreasing graph.

4) The graph of $y = 3^x$ is a reflection of $y = -(3)^x$ in the y-axis.

Algebra 1 CC

Name: _____

## Analyzing the Graph




$-4 \le x \le 6$ except $x = 2$

25. Domain of $f(x)$. $-4 \le x < 2$ and $2 < x \le 6$

26. Range of $f(x)$. $-3 \le y < -2$ and $-2 < y \le 2$ except $y = -2$

27. x-intercepts. $x = \{0, 4\}$

28. minimum value. $y = -3$

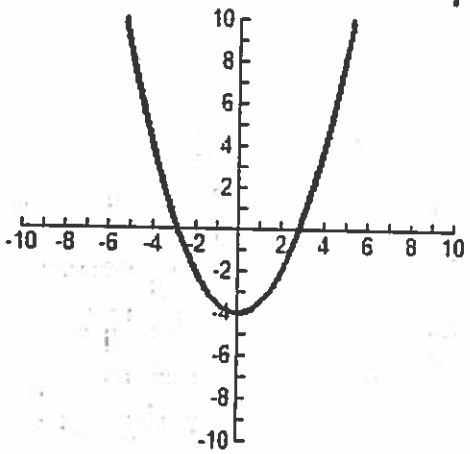
29. maximum value $y = 2$

30. *rate of change between [0, 2]* or $0 \le x \le 2$ → (0, 0), (2, -2)

31. *rate of change between [4, 6]* or $4 \le x \le 6$ (4, 0), (6, 2)

## Analyzing Graphs Practice

17.



**Domain:** all real numbers

**Range:** $y \ge -4$

**x-intercept:** $x = -3$ and $x = 3$ or $x = \{-3, 3$

**y-intercept:** $-4$

**Increasing:** $x \ge 0$

**Decreasing:** $x \le 0$

**Minimum or Maximum:** minimum