Equations and Inequalities - Study Guide

Inequalities

Symbol	Meaning	Graph
$<$	Less than	←○
$>$	Greater than	○→
$\leq$	Less than or equal to At Most Maximum	←●
$\geq$	Greater than or equal to At Least Minimum	●→

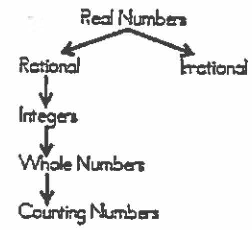
- If you multiply or divide by a negative, FLIP the inequality sign
- Write the variable on the LEFT

The Real Number System

RATIONAL NUMBERS

any number that CAN be written as a fraction

- Counting (1,2,3,...)
- Whole (0,1,2,3,...)
- Integers (...−2,−1,0,1,2...)
- Decimals that repeat (0.333...)
- Decimals that end (−2.5643)
- The square roots of perfect squares ($\sqrt{49}$).
- fractions ($\frac{2}{3}$, $-\frac{7}{2}$, etc.)



rational • rational = rational
rational + irrational = irrational
rational • irrational = irrational

IRRATIONAL NUMBERS

any number that CAN'T be written as a fraction

- Decimals that don't repeat and don't terminate (1.23462...)
- The square roots of non-perfect square ($\sqrt{10}$)

Interval Notation

( ) doesn't include the number
[ ] includes the number

Ex: Integers from (−4,2] would be −3,−2,−1,0,1,2

Solving Equations and Inequalities

1. Distribute (to get rid of parentheses)
2. Combine Like Terms
3. Variables on Both Sides
4. Constant (Add/Subtract)
5. Coefficient (Multiply/Divide)

Cross multiply to solve proportions

Order of Operations

P	Parenthesis/Groupings (absolute value, brackets)
E	Exponents/Square Roots (whichever comes first)
M D	Multiply/Divide (whichever comes first)
A S	Add/Subtract (whichever comes first)

Differences between Equations, Expressions and Equations

- Expressions DO NOT have $=, >, <, \geq, or \leq$ symbols
- Equations ALWAYS have $=$
- Inequalities have $>, <, \geq, or \leq$ symbols
- A TERM is an expression joined by multiplication

EX: x, 2xy, AB (Monomials)
EX: 2x + 1 is 2 TERMS (Binomial)
EX: $3x^2 + 4x - 7$ is 3 TERMS (Trinomial)
*These are all called polynomials

Literal Equations

→Re-writing formulas

- Follow the same steps as solving a regular equation
- Use INVERSE OPERATIONS
- Get variable specified ALONE

EX: Solve for r in terms of V and h.

$V = \pi r^2 h$ divide both sides by πh

$\frac{V}{\pi h} = r^2$ take square root of both sides

$\sqrt{\frac{V}{\pi h}} = r$ final answer ☺

Intepreting Solutions

- If you solve an <u>equation</u> and your solution is a variable equal to a number, you have ONE solution. EX: x = −3
- If you solve an <u>inequality</u>, be careful of what actually is a solution. It must make the inequality statement TRUE. EX: $x < 4$ the number 4 is NOT a solution because 4 is not less than 4 but 3, 2, 1, −50, etc. are all solutions.

Properties of Real Numbers

- The Commutative Property: Addition OR Multiplication
  - Changes Order of the terms
  - $a + b = b + a$ or $a \cdot b = b \cdot a$
- The Associative Property: Addition OR Multiplication
  - Same Order/ Changes groupings
  - $(a + b) + c = a + (b + c)$ or $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
- The Identity Property
  - After the operation, the number (or variable) stays the same
  - Addition – Always 0
    - $a + 0 = a$
  - Multiplication – Always 1
    - $a \cdot 1 = a$
- The Inverse Property
  - Trying to get back to the identity
  - Addition – always the "opposite" sign of a
    - $a + -a = 0$
    - Add to get 0
  - Multiplication – always the reciprocal of a
    - $a \cdot \frac{1}{a} = 1$
    - Multiply to get 1
- The Distributive Property
  - gets rid of parenthesis through multiplication
  - $a(b + c) = ab + ac$ or $a(b - c) = ab - ac$
  - You can also pull out a variable as well (backwards distributing/factoring)
- The Properties of Equality: Addition and Multiplication
  - if you add the same number to both sides of an equation, the sides remain equal
  - if you multiply (or divide) a number to both sides of an equation, the sides remain equal

Solving Word Problems

1. Read and underline key info
2. Define variables with Let Statements/Draw a picture/ Make a table)
3. Write and solve an equation
4. Does your answer make sense?
5. Answer the question

Consecutive Integer: x, x+1, x+2, x+3, …
Consecutive EVEN: x, x+2, x+4, x+6, …
Consecutive ODD: x, x+2, x+4, x+6, …

Perimeter – draw a picture and add all of the sides

Area – draw a picture and use the correct area formula EX: A = LW you would multiply the sides

Pythagorean Theorem – use $a^2 + b^2 = c^2$
For right triangles

- If you don't know 3 things you need 3 LET statements, 2 things 2 LET statements, etc.
- Always define variables first – that will help you get an equation or inequality

Converting Units

- Always write as a form of 1 EX: $\frac{1\ foot}{12\ inches}$
- Look on Reference Sheet for conversions
- Make the units "cross cancel" when converting
- EXAMPLE: Convert 2 ft/sec to inches per minute

$$\frac{2\ ft}{1\ sec} \bullet \frac{12\ in}{1\ ft} \bullet \frac{60\ sec}{1\ min} = 1{,}440\ in/min$$

Functions – Study Guide

Relations and Functions

→Relation – a set of input and output values listed in ordered pairs

- Describes a relationship between two different sets of information

→Function – a relation in which each element of the domain has one and only one element of the range associated with it

- For each x/input/domain there is *one and only one* y/output/range

EXAMPLE: $\{(1,3),(2,5),(6,3),(-2,1)\}$
NON-EXAMPLE: $\{(1,3),(2,5),(1,7)\}$

Function Notation

- $f(x)$ replaces y
- $f(x)$ is the name of the function
- Use substitution to evaluate a function
- x is the input; $f(x)$ is the output

Ordered Pair/Point (x,y)

- On a graph, an ordered pair shows the position on a graph
- First number, x, is the horizontal coordinate $+ \rightarrow, - \leftarrow$
- Second number, y, is the vertical coordinate $+ \uparrow, - \downarrow$

Domain and Range

→Domain

- Left to right
- Input
- x–value
- Independent variable

→Range

- Down to up
- Output
- y–value
- Dependent variable
- $f(x)$

Is it a function?

If it is…

Every x–value has only ONE y–value!!!!!!!!!!!!

The x–value doesn't repeat!

Vertical Line Test

If a vertical line can be drawn through the graph and pass through more than one point, the graph IS NOT a function.

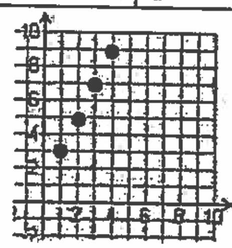
Different Ways to View a Function

EXAMPLE: $f(x) = 2x + 1$ with a domain of $\{1,2,3,4\}$

Table

x	$f(x) = 2x+1$	$f(x)$
1	$f(1) = 2(1)+1$	3
2	$f(2) = 2(2)+1$	5
3	$f(3) = 2(3)+1$	7
4	$f(4) = 2(4)+1$	9

Graph



Translating Functions Basics

Function: $f(x)$

$f(x) + 3$ means…move $f(x)$ up 3

$f(x) - 3$ means…move $f(x)$ down 3

$f(x + 3)$ means…move $f(x)$ left 3

$f(x - 3)$ means…move $f(x)$ right 3

$-f(x)$ means…reflect $f(x)$ over x–axis

$3f(x)$ means… $f(x)$ is narrower

$\frac{1}{3}f(x)$ means… $f(x)$ is wider

Domain Notations

→Interval

( ) for open points/doesn't include

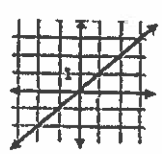
[ ] for closed points/includes

→Inequality

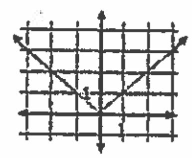
< > for open circle for points/doesn't include; use for ∞ and $-\infty$

≤ ≥ for closed circle for points

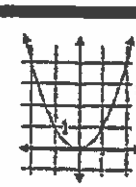
Parent Functions



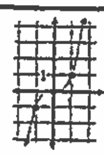
$f(x) = x$
linear



$f(x) = |x|$
absolute value

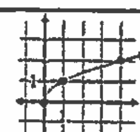


$f(x) = x^2$
quadratic



$f(x) = x^3$
cubic

$f(x) = 2^x$
exponential



$f(x) = \sqrt{x}$
square root

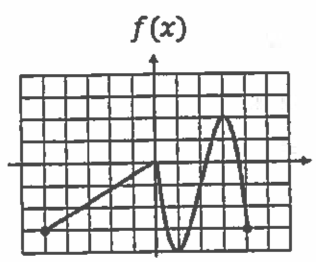
$f(x) = \sqrt[3]{x}$
cube root

Important Vocabulary

<u>maximum</u> – the highest Y–VALUE on a graph
<u>minimum</u> – the lowest Y–VALUE on a graph
<u>domain</u> – the set of all of x–values (LEFT to RIGHT)
<u>range</u> – the set of all of y–values (DOWN to UP)
<u>increasing</u> – the INTERVAL where the X–VALUES go up
<u>decreasing</u> – the INTERVAL where the X–VALUES go down
<u>turning point/vertex</u> – where a graph changes direction; max/min
<u>Zeros/Roots/ x–intercept</u> –where a parabola crosses the x–axis, when y = 0
<u>y–intercept</u> – the point(s) where the graph crosses the y–axis; x–value is 0; the constant in the equation
<u>linear</u> – a graph that makes a straight line
<u>non–linear</u> – a graph that is not a straight line
<u>axis of symmetry</u> – the line that divides a graph into two mirror images of each other; x–value of the vertex; $x = \frac{-b}{2a}$
<u>slope/average rate of change</u> – $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Interpreting a Function

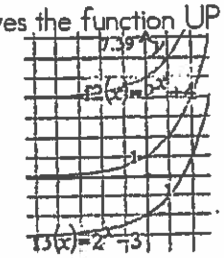
A. Find $f(-5)$
Means… find y when x = −5
ANSWER: y = −3
B. Find all values of x when $f(x) = 2$
Means… find x when y = 2
ANSWER: x = 3
C. What is the maximum of $f(x)$?
Means… find the highest y–value
ANSWER: 2
D. What is the minimum of $f(x)$?
Means… find the lowest y–value
ANSWER: −4



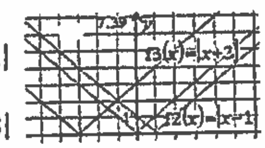
E. What is the domain of $f(x)$ written in inequality notation?
Means… how far left to right does the graph go
ANSWER: $-5 \leq x \leq 4$
F. What is the range of $f(x)$ written in interval notation?
Means… how far down to up does the graph go
ANSWER: $[-4, 2]$
G. What are the turning points of $f(x)$?
Means… where does the graph change from + to – slope or vice versa
ANSWER: (0, 0), (1, −4), and (3, 2)
H. Where is the graph increasing? decreasing?
Means… going from left to right (x), is the graph going up or down
ANSWER: increasing: $-5 < x < 0$ and $1 < x < 3$
decreasing: $0 < x < 1$ and $3 < x < 4$

Translating Functions

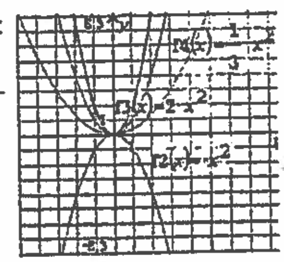
The constant <u>OUTSIDE</u> of the parentheses moves the function UP + or DOWN –
EXAMPLES:
$f(x) = 2^x + 4$
up 4
$f(x) = 2^x - 3$
down 3



The constant <u>INSIDE</u> of the parentheses moves the function LEFT + or RIGHT –
••• opposite of what you think!
EXAMPLES:
$f(x) = |x - 1|$
right 1
$f(x) = |x + 3|$
left 3



The coefficient reflects the function over the x–axis if –, makes wider if between 0 & 1, narrower if bigger than 1
EXAMPLES:
$f(x) = -x^2$
reflects over x–axis
$f(x) = 2x^2$
narrower
$f(x) = \frac{1}{3}x^2$
wider



Other Functions Examples

What is the range of the function $f(x) = 2x + 3$ over the domain $\{0, 1, 2, 3\}$?
ANSWER: $\{3, 5, 7, 9\}$

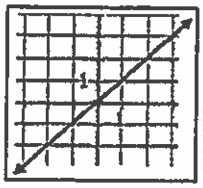
What domain should I use for this situation? A parking garage charges the customer \$2 every half hour.
ANSWER: $\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, \ldots\}$

Given $f(x) = 2x + 1$
→find $f(3)$
ANSWER: $f(3) = 2(3) + 1 = 7$
→find x when $f(x) = 5$
ANSWER: $5 = 2x + 1 \rightarrow x = 2$

Linear Functions - Study Guide

Translating the Parent Function

Parent function: $y = x$



- the constant (the y-intercept) moves the function UP or DOWN
- the coefficient (slope) makes the line STEEPER (bigger than 1) or LESS STEEP (less than 1)
- If the number before the x is NEGATIVE, the line will DECREASE

EX: $y = -2x + 1$ will move the parent function 1 unit up, make it steeper, and will be decreasing

Different Forms of a Line

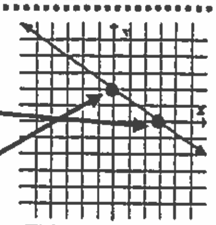
Slope-Intercept: $y = mx + b$
m = slope and b = y-intercept

Standard: $ax + by = c$
Easy to find x & y -intercepts

Point-Slope: $y - y_1 = m(x - x_1)$
(x_1, y_1) is a point and m = slope

Intercepts

- x-intercept: the point where the line crosses the x-axis (when y = 0)
- y-intercept: the point where the line crosses the y-axis (when x = 0)



EX: $2x + 3y = 6$

The x-intercept is:	The y-intercept is:
$2x + 3(0) = 6$ $2x = 6$ $x = 3$	$2(0) + 3y = 6$ $3y = 6$ $y = 2$

Steps to Graphing Lines

1. Solve for y to get into $y = mx + b$
2. identify the slope m and y-intercept b
3. plot the y-intercept on the y-axis
   b = begin
4. use the slope to generate more points
   m = move
   (up +/down – then right +/left –)

When graphing, make sure:

- To plot at least 3 points
- To connect your line with a straightedge
- To draw arrows on both ends of your line
- That your line covers the entire graph
- To label equation & axes

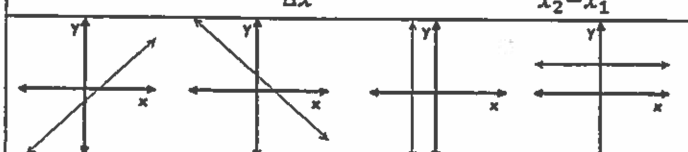
Slope/Average Rate of Change

To find the average rate of change you need TWO POINTS

The steepness of a line

$m = \frac{y\ change}{x\ change}$ OR $m = \frac{rise}{run}$

OR $m = \frac{\Delta y}{\Delta x}$ OR $m = \frac{y_2 - y_1}{x_2 - x_1}$



Positive | Negative | Undefined | Zero

EX: Find the slope of the line $f(x) = \frac{1}{2}x - 7$. $m = \frac{1}{2}$

When in doubt, make a TABLE

EX: Find the slope of a line that passes through the points $f(-2) = 5$ and $f(4) = -1$.

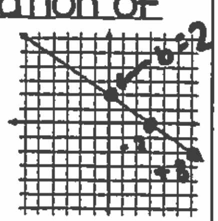
X	-2	4
y	5	-1

+6

-6

$\frac{y\ change}{x\ change} = \frac{-6}{6} = \boxed{-1}$

Finding the Equation of a Line



Find two points
Find their slope (m)
Find the y-intercept (b)
Write the equation in slope-intercept form

EXAMPLE ABOVE: $y = -\frac{2}{3}x + 2$

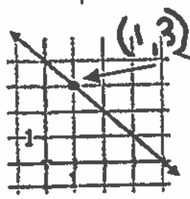
Check in calculator

Is it a solution?

A point is a solution to a linear function if:

- It lies on the graph
- When it is substituted into the equation, it makes a TRUE statement
- If it appears in the table of values

EX: The point (1, 3) is a solution to:



AND

-2	12
0	6
2	0
4	-6

AND

y = −2x + 5 …because
3 = −2(1) + 5
3 = −2 + 5
3 = 3 TRUE!!! ☺

Arithmetic Sequences

Formula given on reference sheet

$$a_n = a_1 + (n-1)d$$

- d is the common difference
- a_1 is the first term in the sequence
- a_n is the n^{th} term in the sequence
- n is a positive integer

EXAMPLE: The third term in an arithmetic sequence is 7 and the sixth term is 19. What is an equation that can be used to find the nth term?

MAKE A TABLE!!!

Term	0	1	2	3	4	5	6
Value	-5	-1	3	7	11	15	19

- common difference/slope is 4
- first term is −1
- y-intercept is −5

Using the formula on the reference sheet:

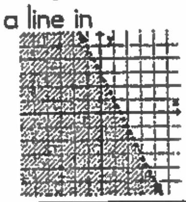
$$a_n = a_1 + (n-1)d$$
$$a_n = -1 + (n-1)4$$

Using slope-intercept form:

$$y = mx + b$$
$$a_n = 4n - 5$$

Graphing Linear Inequalities

1. Graph points like you would a line in y = mx + b form
2. Determine if the line is solid ($\geq$ or $\leq$) or dashed ($>$ or $<$)
3. Shade above the line $>$ or $\geq$
4. Shade below the line $<$ or $\leq$



EX: $y < -2x + 3$
< means dashed and shade below

Solving For "y"

Before you can graph a line or identify the slope/y-intercept, you must solve for y!

EX: Write $3x + 4y = 20$ in slope-intercept form.

$$3x + 4y = 20$$
$$-3x \qquad -3x$$
$$\frac{4y}{4} = \frac{-3x}{4} + \frac{20}{4}$$
$$y = -\frac{3}{4}x + 5$$

Interpreting Solutions to Inequalities

- a point IS a solution if: it is in the shaded area or if it is on a SOLID line
- a point IS NOT a solution if: it is NOT in the shaded area or if it is on a DASHED line

Translating Words into Algebra

- four less than twice x is y → $y = 2x - 4$
- y is equivalent to the sum of half of x and three → $y = \frac{1}{2}x + 3$
- the difference of x and y is 7 → $x - y = 7$
- y is triple the sum of x and 2 → $y = 3(x + 2)$

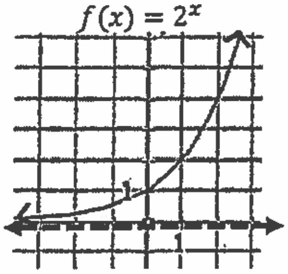
Using the Calculator

Y= (enter equation in Y_1)
Graph to see the graph
2nd graph to see the table of values

Exponential Functions - Study Guide

Parent Function

$f(x) = 2^x$



domain: $-\infty < x < \infty$
range: $y > 0$
y-intercept: (0,1)
increasing: $-\infty < x < \infty$

Evaluating Exponential Functions

EXAMPLE: If $f(x) = 20\left(\frac{1}{2}\right)^x$ find $f(2)$.

→SOLUTION: $f(2) = 20\left(\frac{1}{2}\right)^2 = 20\left(\frac{1}{4}\right) = 5$

So… $f(2) = 5$

…which means $f(x)$ passes through the point (2,5).

Linear versus Exponential

Linear

Look for a common difference
(addition or subtraction of y-values)

+1 +1 +1 +1

x	0	1	2	3	4
y	8	5	2	−1	−4

−3 −3 −3 −3

- Has a constant rate of change:
  Slope $= \frac{\text{y change}}{\text{x change}} = \frac{-3}{1} = -3$
- Has a y-intercept at (0, 8)
- Equation: $y = mx + b$
  m = slope and b = y-intercept
  $y = -3x + 8$

Exponential

Look for a common ratio
(multiplication of y-values)

+1 +1 +1 +1

x	0	1	2	3	4
y	2	6	18	54	162

·3 ·3 ·3 ·3

- Has a common ratio of 3
- 3 is the growth factor because > 1
  Would be decay factor if 0<b<1
- Has y-intercept at (0, 2)
- Equation: $y = a(b)^x$
  a = initial value/y-intercept/x=0
  b = common ratio
  $y = 2(3)^x$

**Check equation in calculator to see if it matches table*

Average Rate of Change

- NEED 2 POINTS
- The SLOPE of a line that passes through two points of a function
- $\frac{y\ change}{x\ change}$

EXAMPLE: Given the function $f(x) = 3(2)^x$, find the average rate of change over the interval $2 \le x \le 4$.

→SOLUTION:

x	f(x)
2	12
4	48

+2 +36

$m = \frac{\Delta y}{\Delta x} = \frac{36}{2} = 18$

Laws of Exponents

- Multiplying with the same base: ADD powers
  $x^a \bullet x^b = x^{a+b}$ EXAMPLE: $5^3 \bullet 5^7 = 5^{10}$
- Dividing with the same base: SUBTRACT powers
  $\frac{x^a}{x^b} = x^{a-b}$ EXAMPLE: $\frac{x^9}{x^5} = x^4$
- Power to power: MULTIPLY powers
  $(x^a)^b = x^{a \cdot b}$ EXAMPLE: $(2^3)^5 = 2^{15}$
- Anything to the FIRST power is ITSELF
  $x^1 = x$ EXAMPLE: $20^1 = 20$
- Anything to the ZERO power is ONE
  $x^0 = 1$ EXAMPLE: $20^0 = 1$
- Negative exponents: re-write as $\frac{1}{}$ the exponent to the positive power (it is not a negative number!!!)
  $b^{-n} = \frac{1}{b^n}$ EXAMPLE: $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

Translating Exponential Functions

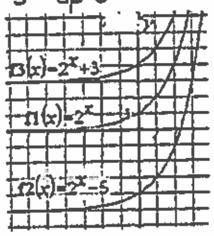
$f(x) = 2^x$ has a y-intercept at (0,1)

The constant OUTSIDE the exponent moves the parent function UP + or DOWN –

EXAMPLES:

$y = 2^x - 5$ down 5

$y = 2^x + 3$ up 3



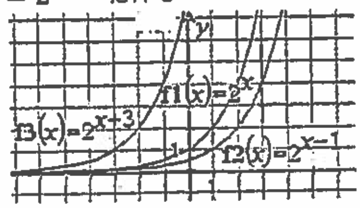
The constant INSIDE the power moves the parent function LEFT + or RIGHT –

*** *opposite of what you think!*

EXAMPLES:

$y = 2^{x-1}$ right 1

$y = 2^{x+3}$ left 3

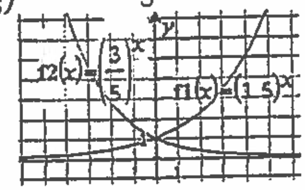


The BASE tells you if the function is INCREASING/GROWTH if $b > 1$
DECREASING/DECAY if $0 < b < 1$

EXAMPLES:

$y = 1.5^x$ increasing

$y = \left(\frac{3}{5}\right)^x$ decreasing



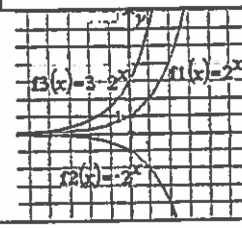
The coefficient

- reflects the function over the x–axis if –
- the value changes the y–int. (must be $y = a(b)^x$ form)

EXAMPLES:

$y = -2^x$ reflects over x–axis

$y = 3(2)^x$ y–intercept at 3



Exponential Growth and Decay

Growth: ADD	Decay: SUBTRACT
$y = a(1+r)^x$	$y = a(1-r)^x$

a = initial amount

r = growth/decay rate as decimal

x = time

GROWTH EXAMPLE:

A population of bugs is growing at a rate of 5% per day. The initial population is 22 bugs. Find a formula that models this situation.

→SOLUTION:

100% + 5% = 105% as a decimal 1.05

So... $y = 22(1.05)^x$

initial amount growth factor

DECAY EXAMPLE:

A radioactive material that is initially 55 grams decays at a rate of 14% per day. Find a formula that models this situation.

→SOLUTION:

100% – 14% = 86% as a decimal is 0.86

So... $y = 55(0.86)^x$

Geometric Sequences

Sequence means MAKE A TABLE

$a_n = a_1 r^{n-1}$ ← Given on reference sheet

a_n = the n^{th} term/any term

n = term number

a_1 = initial value

r = common ratio (growth/decay factor)

EXAMPLE:

Write a formula that can be used to find the n^{th} term of the sequence:

20, 10, 5, 2.5, ...

→SOLUTION:

term	1	2	3	4
#	20	10	5	2.5

$\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$

$a_n = 20\left(\frac{1}{2}\right)^{n-1}$

•Percent Tips•

- convert to a decimal → move decimal 2 units left
- Increase means ADD to 100%
- Decrease means SUBTRACT from 100%

Polynomials - Study Guide

Important Vocabulary

Term – A number, variable or product of numbers and variables

Like/similar terms – Two or more terms that have the same variable(s) with the same exponents

Unlike terms – Two or more terms with different variables and/or different powers

Monomial – A single variable or number, or a product of a coefficient and one or more variables with exponents that are whole numbers

Binomial – A polynomial with two terms

Trinomial – A polynomial with three terms

Simplest form – When a polynomial contains no like terms

Standard form – When a polynomial is written with the exponents in descending order (high to low exponents)

Coefficient – The number being multiplied to the variable (written in front of variable)

Leading coefficient – The coefficient of the variable with the highest exponent

Constant – A number that is not attached to any variable

Variable – A letter or symbol used to replace a number

Laws of Exponents

- ⇨ Anything to the FIRST power is ITSELF
  $20^1 = 20$
- ⇨ Anything to the ZERO power is ONE
  $20^0 = 1$
- ⇨ When MULTIPLYING exponents that have the same base, ADD the powers
  $x^2 \bullet x^4 = x^6$
- ⇨ When DIVIDING exponents that have the same base, SUBTRACT the powers
  $x^{10} \div x^8 = x^2$
- ⇨ When raising an exponent to another power, MULTIPLY the powers.
  $(x^2)^4 = x^8$

Polynomial

An expression made up of variables and constants.

Adding/Subtracting Polynomials

1. Keep 1st polynomial
2. Distribute sign to 2nd polynomial
3. Combine like terms (same variable & exponent)
4. Add/Subtract Coefficients & Keep variable/exponent

Tips:

- Variable/exponent NEVER changes
- Write in hidden coefficients of 1
  EX: $2x + x = 2x + 1x = 3x$, not $2x$
- Be careful when distributing a negative
  EX: $(3x + 1) - (2x - 4) = 3x + 1 - 2x + 4$
- x subtracted from y looks like $y - x$ (switch order)
- SUM or TOTAL means ADD
- DIFFERENCE means SUBTRACT

Multiplying Polynomials

1. Write in hidden exponents and coefficients of 1
2. Distribute (if necessary)
3. Multiply coefficients
4. Keep variable & Add exponents

Tips:

- Use the LAWS OF EXPONENTS
- Powers ALMOST ALWAYS change
- PRODUCT means MULTIPLY
- Expand whenever you see a power
  EX: $(5x)^3 = 5x \bullet 5x \bullet 5x = 125x^3$
  EX: $(x+2)^2 = (x+2)(x+2)$ Double distribute!
- AREA and VOLUME means multiply

Adding vs. Multiplying

Simplify: $3x^2 + 5x^2$ $= 8x^2$ Add coefficients Powers don't change	Simplify: $3x^2 \bullet 5x^2$ $= 15x^4$ Multiply coefficients Add powers

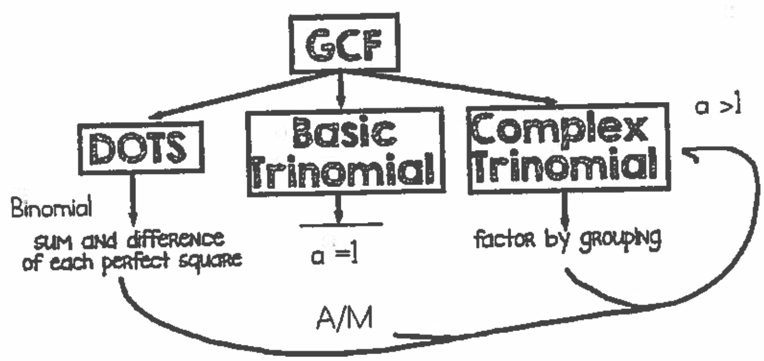
Double Distributing (and beyond)

Be careful of INTEGER RULES!

(binomial)(binomial)	$(\text{binomial})^2$	(binomial)(trinomial)
$(x+3)(x-7)$ $x^2-7x+3x-21$ $x^2-4x-21$	$(x-5)^2$ $(x-5)(x-5)$ $x^2-5x-5x+25$ $x^2-10x+25$	$(x-1)(x^2+3x-2)$ $x^3+3x^2-2x-1x^2-3x+2$ x^3+2x^2-5x+2

Factoring Completely

- Remember: we always RE-WRITE and NEVER change the value of an expression
- Always check by multiplying back through to make sure you get what you started with



REPEAT UNTIL FACTORED

Examples:

$x^2+4x-45$ A: 4 M: −45 9,−5 $(x+9)(x-5)$	x^4+7x^2+10 A: 7 M: 10 5,2 $(x^2+5)(x^2+2)$
b^2-25 ★DOTS $b\cdot b=b^2$ ✓ $5\cdot 5=25$ ✓ subtract ✓ $(b+5)(b-5)$	$2m^3-26m^2+72m$ GCF: 2m $2m(m^2-13m+36)$ $2m(m-9)(m-4)$ A: −13 M: 36 −9,−4
$18a^2-33a+9$ GCF: 3 $3(6a^2-11a+3)$ S | P: −11 | 18 ($6\cdot 3$) $-9\cdot -2=18$ ✓ $-9+-2=-11$ ✓ $3(6a^2-9a-2a+3)$ $3[3a(2a-3)-1(2a-3)]$ $3(3a-1)(2a-3)$	x^4-81 DOTS $x^2\cdot x^2=x^4$ ✓ $9\cdot 9=81$ ✓ $(x^2+9)(x^2-9)$ DOTS $x\cdot x=x^2$ ✓ $3\cdot 3=9$ ✓ $(x^2+9)(x+3)(x-3)$

Quadratic Functions - Study Guide

Important Vocabulary

standard form: $y = ax^2 + bx + c$
leading coefficient: the "a" in standard form
roots/zeros/x-intercepts/solutions: where parabola crosses x-axis, when y = 0
vertex: the turning point of the parabola: max/min
intercept: the point where the graph crosses the y-axis, when x = 0
axis of symmetry: the x-value of the vertex; x =
maximum: the highest y-value
minimum: the lowest y-value
concave up: vertex is a minimum ∪
concave down: vertex is a maximum ∩

Graphing on the TI-83/84

- Put equation in y= form and enter in Calculator
- 2nd, Calc, Min/Max to find turning point/vertex

  2nd, Calc, 2: Zeros to find roots/solutions

Simplifying Radicals

1. Find the BIGGEST perfect square factor
2. Write as the product of 2 radicals (make sure the perfect square is first)
3. Evaluate the perfect square

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

EXAMPLE: Simplify $x = 3\sqrt{48}$

$$x = 3 \bullet \sqrt{16} \bullet \sqrt{3}$$
$$x = 3 \bullet 4 \bullet \sqrt{3}$$
$$x = 12\sqrt{3}$$

Factoring REVIEW

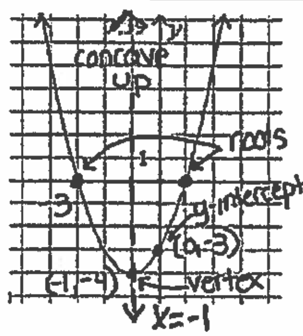
Factor!
Try GCF first! Then try:

DOTS if binomial — **A/M or grouping** if trinomial

REPEAT UNTIL FACTORED COMPLETELY

Different Forms of a Parabola

FORM	TELLS US	EXAMPLE
Standard Form	ax^2 • Opens: UP: a is + DOWN: a is − • NARROW: $a > 1$ WIDE: $0 < a < 1$ • y-intercept: is the CONSTANT	$y = x^2 + 2x - 3$ ⇨ opens up b/c x^2 is positive ⇨ y-intercept at (0, −3)
Vertex Form	$a(x-h)^2 + k$ • Opens: UP: a is + DOWN: a is − • NARROW: $a > 1$ WIDE: $0 < a < 1$ • Vertex: $(-h, k)$ h is always OPPOSITE SIGN	$y = (x+1)^2 - 4$ ⇨ opens up b/c number in front of parenthesis is positive ⇨ vertex: (−1, −4)
Factored Form	• roots/zeros set each factor =0	$y = (x-1)(x+3)$ ⇨ roots at x = 1 and x = −3



The SAME function written 3 different ways!
$y = x^2 + 2x - 3$
$y = (x+1)^2 - 4$
$y = (x-1)(x+3)$

axis of symmetry: $x = -\frac{b}{2a}$
the x-value of the vertex: x =

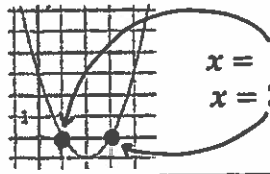
Vertex Form

$f(x) = -x^2 + 8x + 9$
$\quad -9 \qquad\qquad -9$
$f(x) - 9 = -x^2 + 8x$
$f(x) - 9 = -1(x^2 + 8x)$
$f(x) - 9 = -1(x^2 + 8x + 16)$
$f(x) - 9 + 16(-1) = -1(x^2 + 8x + 16)$
$f(x) - 25 = -1(x+4)^2$
$\quad +25 \qquad +25$
$\boxed{f(x) = -(x+4)^2 + 25}$

Vertex (-4, 25)

Steps when solving for Vertex Form:
1) Move the constant term to the left hand side.
2) If a = 1, skip to step 3.
   If a ≠ 1:
   Factor out the "a" value on the right hand side
3) Add the needed value to create a perfect square $\left(\frac{b}{2}\right)^2$ trinomial to right hand side *inside* the parenthesis
4) Add the same value to the left hand side and also multiply that value by the "a" value
5) Left side: Combine like terms
   Right side: Factor the perfect square trinomial
6) Solve for y *Vertex Form*
7) Identify the Vertex (h,k)

Solving Quadratic Equations ~ Finding Roots/Zeros

	Steps	Examples
Graphically	Graph to find the x-intercepts/roots ***2nd, Calc, 2: zero** Do this for EVERY root	$y = x^2 - 4x + 3$  $x = 1$ $x = 3$
Factoring	1. Factor 2. Set each factor = 0 3. Solve to find roots	$y = x^2 + 7x - 18$ SET = 0 !!! $0 = x^2 + 7x - 18$ $0 = (x+9)(x-2)$ $0 = x + 9 \quad 0 = x - 2$ $x = -9 \quad x = 2$
Inverse Operations	1. Move constant to other side 2. Take square root of both sides (±!!!!) ***use when there isn't a "bx" term	$y = x^2 - 25$ SET = 0 !!! $0 = x^2 - 25$ $25 = x^2$ $x = \sqrt{25}$ $x = \pm 5$ so... $x = -5$ and $x = 5$
Completing the Square	**Steps:** **1) Be sure that the leading coefficient is 1. If not, divide each term by that value to create a leading coefficient of 1.** **2) Move the constant term to the right hand side.** **3) Add the needed value to create a perfect square trinomial to both sides of the equation so that it is balanced.** ****To find this value, do*** $\left(\frac{b}{2}\right)^2$. $\boxed{\left(\frac{8}{2}\right)^2 \; 4^2 \; 16}$ **4) Factor the perfect square trinomial** **5) Take the square root of each side & solve.** ****Remember to consider both positive & negative results & to simplify the radical!***	$x^2 + 8x - 4 = 0$ $+4 \quad +4$ $x^2 + 8x = 4$ $x^2 + 8x + 16 = 4 + 16$ $x^2 + 8x + 16 = 20$ $(x+4)^2 = 20$ $x + 4 = \pm\sqrt{20}$ $x + 4 = \pm\sqrt{4}\sqrt{5}$ $x + 4 = \pm 2\sqrt{5}$ $-4 \quad -4$ $\boxed{x = -4 \pm 2\sqrt{5}}$
Quadratic Formula	1. Identify a, b and c 2. Substitute and solve ***use only if you CAN'T factor or they want solutions as a decimal or simplest radical form.	$x^2 - 5x + 5 = 0$ $ax^2 + bx + c = 0$ SET = 0 !!! $a = 1$ $b = -5$ $c = 5$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(5)}}{2(1)}$ $\boxed{x = \frac{5 \pm \sqrt{5}}{2}}$

Solving: Linear versus Quadratic

Linear: get x alone	Quadratic (x^2): set = 0
$5x - 8 + 3x = 6(x - 1)$	$x^2 - 7 = x + 5$
$5x - 8 + 3x = 6x - 6$	$x^2 - x - 7 - 5 = 0$
$5x + 3x - 6x = 8 - 6$	$x^2 - x - 12 = 0$
$\frac{-1x}{-1} = \frac{2}{-1}$	$(x-4)(x+3) = 0$
$\boxed{x = -2}$	$x - 4 = 0$, $\boxed{x = 4}$ | $x + 3 = 0$, $\boxed{x = -3}$

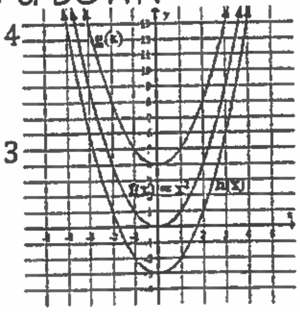
Transforming Parabolas

OUTSIDE parenthesis (constant):

UP + or DOWN –

$g(x) = x^2 + 4$
up 4

$h(x) = x^2 - 3$
down 3



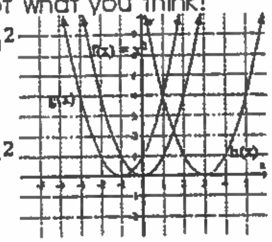
INSIDE parenthesis

LEFT + or RIGHT –

** opposite of what you think! **

$g(x) = (x+1)^2$
left 1

$h(x) = (x-3)^2$
right 3



IN FRONT

reflects over the x-axis if –

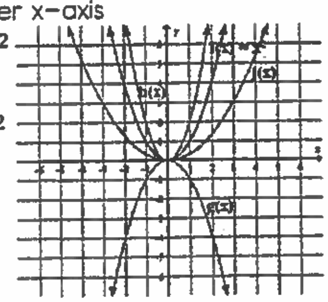
wider if a fraction smaller than 1

narrower if bigger than 1

$g(x) = -x^2$
reflects over x-axis

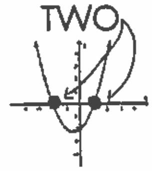
$h(x) = 2x^2$
narrower

$j(x) = \frac{1}{3}x^2$
wider

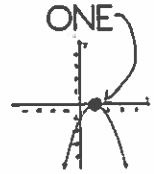


How many roots?

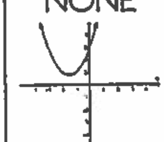
TWO



ONE



NONE



Quadratic Formula

In the quadratic $0 = ax^2 + bx + c$, the roots can be found using...

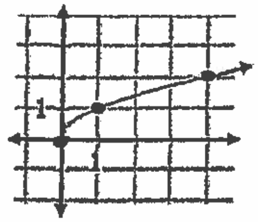
$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

On ref. sheet

Special Functions – Study Guide

Square Root Parent Function

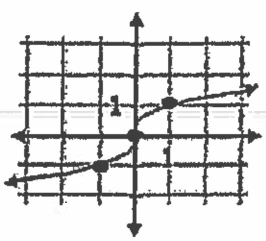
$f(x) = \sqrt{x}$



Enter in calc: y= 2nd, x^2
domain: $x \geq 0$
range: $y \geq 0$
y–intercept: y = 0
zero(s): x = 0
increasing: $x > 0$
decreasing: never

Cube Root Parent Function

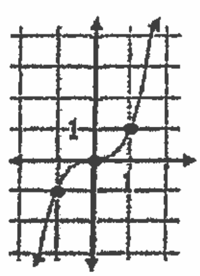
$f(x) = \sqrt[3]{x}$



Enter in calc: y= Math, 4, x
domain: $-\infty < x < \infty$
range: : $-\infty < y < \infty$ y–
intercept: y = 0
zero(s): x = 0 a
increasing: always
decreasing: never

Cubic Parent Function

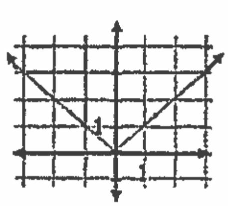
$f(x) = x^3$



Enter in calc: y= x^3
domain: $-\infty < x < \infty$
range: : $-\infty < y < \infty$
y–intercept: x = 0
zero(s): y = 0
increasing: always
decreasing: never

Absolute Value Parent Function

$f(x) = |x|$ Enter in calc: y= Math, Num, 1: abs
domain: $-\infty < x < \infty$



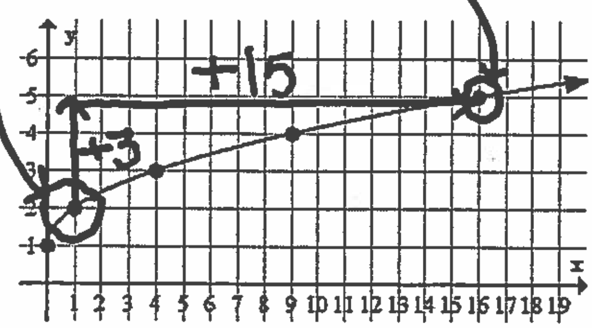
range: $y \geq 0$
y–intercept: y = 0
zero(s): x = 0
increasing: $x > 0$
decreasing: $x < 0$

Average Rate of Change

- MAKE A TABLE!!!
- Need: TWO POINTS
- Find the points from a table or graph
- The SLOPE of a line that passes through TWO POINTS of a function
- $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

EX/
Given the function $f(x) = \sqrt{x} + 1$, find the average rate of change over the interval $1 \leq x \leq 16$

→SOLUTION:



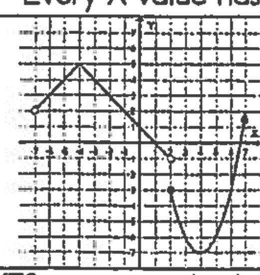
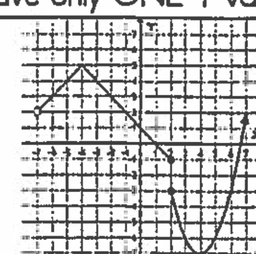
X	Y
1	2
16	5

+15, +3

So... $\frac{3}{15} = \frac{1}{5}$

Vertical Line Test/Is it a Function?

- Every X value has to have only ONE Y value

	
YES: every x–value has exactly one y–value	NO: the x–value of 2 has two y–values: – 1 and –3

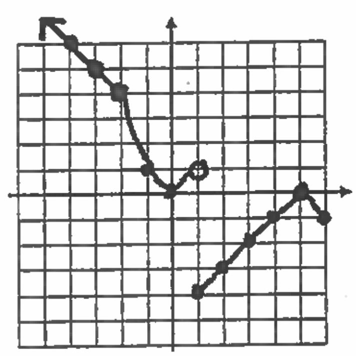
Piecewise Functions

⇨ Be careful of
- open points
  - does NOT include
  - $<$ or $>$
- closed points
  - DOES include
  - $\leq$ or $\geq$

⇨ Can be linear or non-linear

EX: Graph the function:

$$f(x) = \begin{cases} -x+2 & -\infty < x \leq -2 \\ x^2 & -2 < x < 1 \\ |x-5| & 1 \leq x \leq 6 \end{cases}$$



Then find:

$f(1) =$ −4

$f(-2) =$ 4

$f(5) =$ 0

$f(0) =$ 0

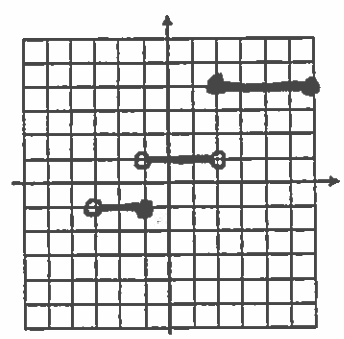
Step Functions

⇨ Be careful of
- open points
  - does NOT include
  - $<$ or $>$
- closed points
  - DOES include
  - $\leq$ or $\geq$

⇨ horizontal lines that look like steps

EX: Find the equation of the function.

$$f(x) = \begin{cases} -1 & -3 < x \leq -1 \\ 1 & -1 < x < 2 \\ 4 & 2 \leq x \leq 6 \end{cases}$$



Then find:

$f(2) =$ 4

$f(4.6) =$ 4

$f(-1) =$ −1

Evaluating Functions Tips

Absolute Values	Exponents
– The distance a number is from zero – EX: Find y if $y = \lvert x+5 \rvert$ when $x = -9$ $y = \lvert x+5 \rvert = \lvert -9+5 \rvert = \lvert -4 \rvert = 4$ So $y = 4$ or the point (−9, 4)	– When substituting, always use parentheses – EX: Find y if $y = x^2 - 1$ when $x = -3$ $y = x^3 - 1 = (-3)^2 - 1 = 9 - 1 = 8$ So $y = 8$ or the point (−3, 8)
Square Roots	**Cube Roots**
– CAN'T take the square root of a negative – EX: Find y if $y = \sqrt{x} - 3$ when $x = 25$ $y = \sqrt{x} - 3 = \sqrt{25} - 3 = 5 - 3 = 2$ So $y = 2$ or the point (25, 2)	– CAN take the square root of a negative – EX: Find y if $y = \sqrt[3]{x} + 1$ when $x = -8$ $y = \sqrt[3]{x} + 1 = \sqrt[3]{-8} + 1 = -2 + 1 = -1$ So $y = -1$ or the point (−8, −1)

Systems - Study Guide

Solving Linear Systems

GRAPHICALLY

Steps

1. Graph and label both equations
2. Find where the 2 lines intersect
3. Label point of intersection
4. Check by graphing on calculator

Using the TI-83/84

1. Put both lines in y= form
2. Enter both in calculator
3. 2nd, Calc, Int, Enter 3 times
4. Check in your table!

Example

$$y - x = 2$$
$$y = -2x + 5$$

$y - x = 2$

$y = 1x + 2$

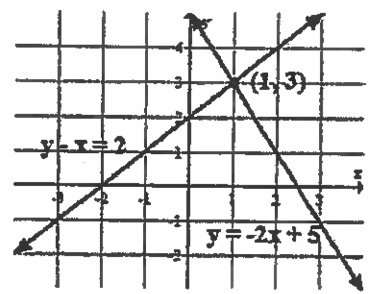
Start @ $(0,2)$

$\frac{1\uparrow}{1\rightarrow}$

$y = -2x + 5$

start @ $(0,5)$

$\frac{-2\downarrow}{1\rightarrow}$



Solution: $(1,3)$

SUBSTITUTION

Steps

1. Solve for a variable (either x or y or both)
2. Substitute
3. Solve for the remaining variable
4. Substitute your new value back into one of the equations then solve to get the other variable
5. Write answer as an ordered pair.
6. Check!

Example

$$y - x = 3$$
$$x + y = 7$$

$y - x = 3$

$y = \boxed{x + 3}$

$x + y = 7$

$x + (x + 3) = 7$

$2x + 3 = 7$

$-3 \quad -3$

$\frac{2x}{2} = \frac{4}{2}$

$\boxed{x = 2}$

$x + y = 7$

$2 + y = 7$

$-2 \quad -2$

$\boxed{y = 5}$

Solution: $(2,5)$

ELIMINATION

Steps

1. Sort to line up the variables and equal signs
2. Multiply one or both equations by a constant to create the additive inverse of one of the variables
3. Add or subtract both equations to eliminate one of the variables (the one that has the additive inverse)
4. Solve for the variable that remains
5. Substitute your new value back into one of the equations (it doesn't matter which one so pick the easier one) then solve to get the other variable
6. Write answer as an ordered pair.
7. Check!

Example

$$2(3x - y = 17) \rightarrow 6x - 2y = 34$$
$$2x + 2y = 14 \qquad + 2x + 2y = 14$$

$\frac{8x}{8} = \frac{48}{8}$

$\boxed{x = 6}$

$3x - y = 17$

$3(6) - y = 17$

$18 - y = 17$

$-18 \quad -18$

$\frac{-y}{-1} = \frac{-1}{-1}$

$\boxed{y = 1}$

Solution: $(6,1)$

Solving Linear/Quadratic Systems

Algebraically	Graphically
Solve using SUBSTITUTION ▪ set them both = y then = to each other ▪ set one = y and substitute y into the other	Graph both and find the points of intersection ▪ line: write in y = mx + b form ▪ quadratic: use table of values

2. **Solve this system of equations graphically AND algebraically.**

$y = -x^2 + 2x + 4$

$x + y = 4$

$x+y=4$ $\quad m=\frac{-1}{1}$ $\quad b=4$

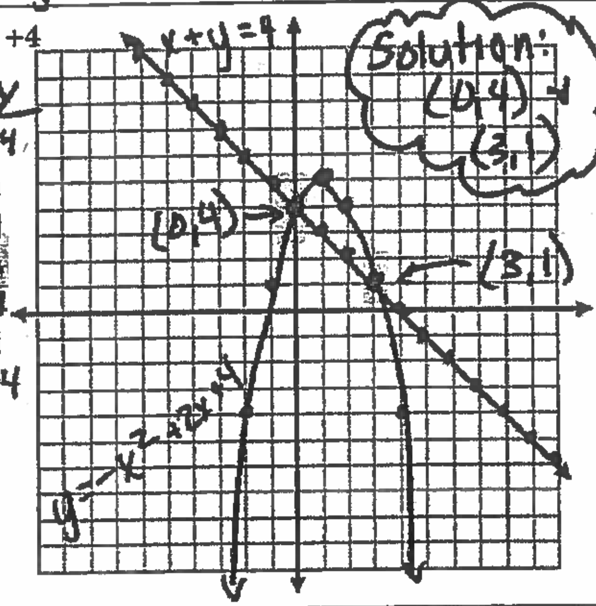
$y=-x+4$

$y=4-x$

GRAPHICALLY

$y=-x^2+2x+4$

x	y
-2	-4
-1	1
0	4
1	5
2	4
3	1
4	-4



ALGEBRAICALLY

$-x^2+2x+4=4-x$

$\underline{+x^2\ -2x\ -4}\quad \underline{-4\ -2x\ +x^2}$

$0=x^2-3x$

$0=x(x-3)$

$0=x \qquad x-3=0$

$\qquad\qquad x=3$

$0+y=4 \qquad 3+y=4$

$y=4 \qquad y=1$

Solution: (0,4) + (3,1)

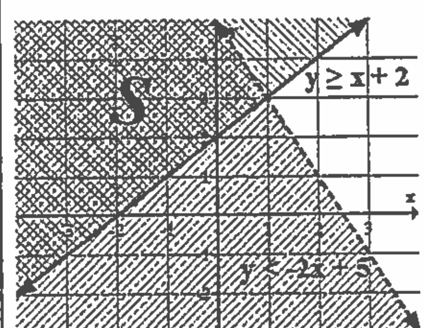
Solving a System of Linear Inequalities

Steps:

1. Graph and label both inequalities
2. Find where the shaded regions overlap and label with an "S"

- Solutions are in the overlapping region (NOT on dashed lines, so if the point of intersection is on a dashed line, it is NOT a solution!)
- Shading on calc: go to y=, go left two times and Enter to change line shading up > or down <

Example:



- (1, 3) IS NOT a solution because one of the lines is dashed
- (−2, 3) IS a solution because it is in the shaded region of both inequalities

Real Life Applications of Systems

General steps:	Things to be careful of:
1. Define any unknowns and write them as variables (Let x = and y =) 2. Find two equations or inequalities that model the situation 3. Use your methods of solving to find the values of both variables	– Did they restrict the domain? – Does your answer make sense? – Did you use correct units?

Statistics - Study Guide

Box Plots

minimum MinX | quartile 1 Q_1/25th Percentile | median Median/50thP | quartile 3 Q_3/75th P. | maximum MaxX

Using the TI-83/84

1. Stat, Edit, L_1
2. Stat, Calc, 1: 1-Var Stats
3. Enter and scroll down
4. Write down MinX, Q1, Med, Q3, MaxX
5. Create an even interval scale on number line
6. Plot 5 values above number line and create box with 3 Quartile values and whiskers to the Min and Max

Variation in a Data Set

- Use One-Variable Statistics

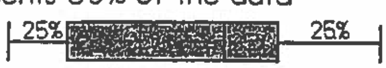
→ Standard Deviation

- Tells us how far a data point is away from the mean
- The farther apart the data, the bigger the SD

Use Sx from calculator

→ Interquartile Range (IQR)

- (quartile 3) – (quartile 1)
- Represents 50% of the data



Types of Data

One Variable Data

- Data with 1 variable
- Can be modeled using a histogram, box plot, or dot plot

Two Variable (Bivariate) Data

- Data with 2 variables
- Typically modeled using a scatterplot
- Use line of best fit
- Correlation – relationship
- Causal relationship – Does one CAUSE the other?

Finding One-Variable Statistics

Using the TI-83/84

1. Stat, Edit, L_1
2. Stat, Calc, 1: 1-Var Stats
3. Enter and scroll down

$\bar{x}$ = mean

Sx = standard deviation

MinX = minimum

Q_1 = quartile 1/25th Percentile

Med = median

Q_3 = quartile 3/75th Percentile

MaxX = maximum

Measures of Central Tendency

- Use One-Variable Statistics
- A single number used to describe a set of data as a whole
- Most common: MEAN and MEDIAN

Mean – average

Median – middle value

Mode – value that appears the most

*not shown on calc.

Interpreting Data

- Outliers – a value that "lies outside" most of the other values in a set of data
- Best measure of center - mean unless there is an outlier then use the median

Regressions/Line of Best Fit

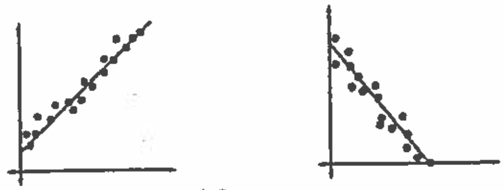
Regression – a line that best represents the data on a scatter plot (linear, quadratic, exponential)

Using the TI-83/84

1. TYPE YOUR DATA INTO LISTS
   Stat, Edit, L1 and L2
2. Stat, Calc, 4:LinReg (ax+b)
   OR
   Stat, Calc, 0:ExpReg (ab^x)
3. Write equation by substituting values in y= form
4. FIND CORRELATION COEFFICIENT
   2nd, 0, Diagnostics On, Enter two times
   Stat, Calc, 4:LinReg (ax+b)
   Write r value follwoing rounding instructions
5. The closer r is to 1 or -1 it suggests a stronger positive/ negative correlation

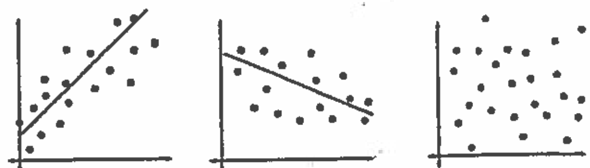
Correlation Coefficient: r

- Tells us how well the function matches the data
- ALWAYS between –1 and 1
- Use your calculator to find r



strong positive: good fit $r = 0.93$

strong negative: good fit $r = -0.9$



weak positive bad fit $r = 0.6$

weak negative bad fit $r = 0.5$

no correlation terrible fit $r = 0$

Two-Way Frequency Tables

EXAMPLE:

Gender	Favorite Color			
	Blue	Red	Green	TOTAL
Male	2	6	1	9
Female	3	2	8	13
TOTAL	5	8	9	22

- Always find totals if they don't give them to you
- Be careful of how the question is worded

Joint Frequency: Each individual cell value

Marginal Frequency: The total for a row or column

Conditional Frequency: comparing a joint to marginal frequency

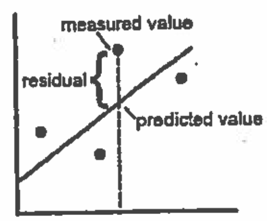
Relative Frequency: The cell value over the grand total

1. What is the conditional frequency of females who state red as their favorite color?

$$\frac{2}{8} = 0.25 = 25\%$$

Residuals

- MEASURED – PREDICTED = RESIDUAL
- GOOD: evenly spaced above/below the x-axis
- BAD: see a pattern



good model ☺ bad model ☹

Cumulative Frequency

Histograms

Weights of adults:

Interval	Frequency	Cumulative Frequency	
51-100	7	7	51-100
101-150	7	14	51-150
151-200	10	24	51-200
201-250	6	30	51-250